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RANSAC-Based Fault Detection and Exclusion Algorithm for Single-Difference Tightly Coupled GNSS/INS Integration

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5 Abstract—There is an urgent need for high-accuracy and highreliability navigation and positioning in life safety fields such as 6 7 intelligent transportation and automotive driving, especially in complex urban environments. Although, compared with the GNSS 8 and loosely coupled integration, a tightly coupled GNSS/INS inte-9 gration can improve the positioning reliability by using raw obser-10 vations, it still suffers from external challenging environments such 11 as the multipath effect. Therefore, the fault detection algorithm is 12 a premise and guarantee to realize quality control of GNSS/INS 13 integration. Inspired by the application of the random sample con-14 sensus (RANSAC) algorithm in GNSS fault detection, this article 15 16 proposes a RANSAC-based fault detection and exclusion algorithm for single-difference tightly coupled GNSS/INS integration. Here, 17 18 a between-receiver single-difference (BRSD) model was designed to prevent the consumption of GNSS observations and reduce the 19 waste of effective parameters, and the global proportion statistics 20 of faults were introduced into the typical RANSAC algorithm to 21 further ensure detection reliability. In this study, the effect of the 22 23 main parameters on the proposed detection algorithm was analyzed and verified by artificial cycle slips. Multiple filed tests, including 24 typical urban scenarios, were conducted to verify the feasibility 25 26 and effectiveness of the proposed method. The comprehensive test results show that the north and east positioning accuracy in 27 terms of cumulative distribution function (CDF, CDF = 95%) are 28 29 improved by 45% and 42% over the tightly coupled mode without the proposed detection method. 30

Index Terms—Fault detection, RANSAC, tightly coupled,
 between-receiver single difference, GNSS/INS integration.

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I. INTRODUCTION

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HE integration of the global navigation satellite system 34 (GNSS) with an inertial navigation system (INS) can 35 achieve complementary advantages, providing pose services 36 with high accuracy and continuity for the intelligent vehicle nav-37 igation and control. There has been an increasing demand for the 38 positioning accuracy and reliability of GNSS/INS integration, 39 especially using low-cost sensors (e.g., microelectromechanical 40 system (MEMS) inertial measurement unit (IMU)), in safety of 41 life applications such as intelligent driving [1], [2]. However, 42 complex urban environments bring severe challenges to GNSS 43 observation. For example, satellite visibility is completely or 44 partially obscured in urban environments, which results in a 45 decrease in GNSS positioning accuracy and continuity [3], [4]. 46

Tightly coupled (TC) GNSS/INS integration can directly 47 utilize raw GNSS observations for measurement updates and 48 performs better than loosely coupled (LC) integration in areas 49 with partially blocked GNSS access [5]. Although GNSS/INS 50 integration can ensure positioning continuity, satellite signals 51 are still interfered by the non-line-of-sight (NLOS) signals and 52 multipath effects, resulting in GNSS observation faults and 53 ultimately affecting the positioning accuracy and reliability 54 in challenging environments. Therefore, quality control is a 55 prerequisite to correctly detect faults and improve positioning 56 accuracy and reliability. Common GNSS/INS integration fault 57 detection methods are conducted by constructing test statistics 58 based on the innovation vector of a Kalman filter [6], [7]. These 59 methods apply quality control at the information fusion level 60 and are not effective for multiple faults detection. Classical 61 receiver autonomous integrity monitoring (RAIM) algorithms 62 have been developed to provide fault detection and exclusion 63 (FDE) [8], [9], but they generally work properly in the case of a 64 single fault and cannot provide reliable multiple faults detection 65 capabilities. Although there are some methods such as multiple 66 hypothesis solution separation (MHSS) and an advanced RAIM 67 (ARAIM) method to solving multiple faults, these methods will 68 be ineffective in presence of significantly large biases or large 69 proportion of faulty satellites [10], [11]. 70

Random sample consensus (RANSAC) can achieve correct71GNSS fault detection in cases of multiple and small faults, and72it is the research hotspot of GNSS fault detection and exclusion73[12]. RANSAC is an iterative method to estimate the parameters74of a mathematical model from a set of observed data that contains75

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faults, and it can be interpreted as a fault detection method. The 76 RANSAC algorithm was first proposed by Fischler and Bolles 77 [13] and has been widely used in the field of computer vision 78 79 and is capable of interpreting or smoothing data containing a significant percentage of faults [14]. Schroth et al. [15] first 80 proposed the range consensus (RANCO) algorithm and the 81 suggestion range consensus (S-RANCO) algorithm to detect 82 faulty GNSS range measurements based on the elementary idea 83 of the RANSAC algorithm. Furthermore, Schroth et al. [16] 84 85 optimized the performance of RANCO by enhancing the subset evaluation, the subset selection algorithm and the modified 86 threshold definition to significantly reduce the missed detection 87 rate and false alarm rate. 88

On the basis of Schroth's research work, many performance 89 (in terms of accuracy, effectiveness, and stability) improve-90 ment methods have been studied. Groves and Jiang et al. [17], 91 [18] applied weighting based on consistency and C/N_0 to the 92 common RANSAC cost function to reduce the number of the 93 largest GNSS faults and used four GNSS measurements plus a 94 height-aiding measurement instead of 5 GNSS measurements 95 96 to improve the positioning accuracy. Su et al. [19] proposed a fast RANSAC algorithm using geometric dilution of precision 97 (GDOP), the line-of-sight (LOS) vector and singular value de-98 composition (SVD) for subset preselection to solve the large 99 100 computational load problem in the traditional RANSAC algorithm. An augmented version of the RANSAC algorithm that 101 performs a final range comparison using the state estimate ob-102 tained with only the inliers identified by RANSAC was proposed 103 for more reliable availability [20]. Zhao et al. [21] proposed a 104 modified RANCO algorithm based on a genetic algorithm to 105 106 inhibit the amount of exponential calculation. In addition, the RANSAC algorithm was introduced to protect the robustness 107 and accuracy of a multi-GNSS time-difference carrier phase 108 (TDCP) solution [22]. 109

Currently, the RANSAC algorithm is applied to the fault de-110 tection and exclusion of individual GNSS range measurements, 111 and the relevant research focuses on improving the compu-112 tational efficiency and fault identification precision. Although 113 some research has utilized the RANSAC algorithm to address the 114 115 issue of loosely coupled GNSS/INS integration as demonstrated in some studies [23], [24], a critical unresolved problem pertains 116 to the minimum number of satellites required in subset construc-117 tion. This issue remains unsolved, and it is still necessary to use 118 a minimum of four satellites. The existing relevant research does 119 not design the RANSAC-based algorithm in the tightly coupled 120 GNSS/INS integration, and not fully play the auxiliary role of 121 inertial navigation information in the subset construction. 122

Inspired by its application to GNSS positioning solu-123 tions, RANSAC is applied to single-difference tightly coupled 124 GNSS/INS integration for robust and high-accuracy positioning 125 in this study. The characteristics and contribution of RANSAC-126 based fault detection in the context of single-difference tightly 127 coupled GNSS/INS integrated navigation can be summarized as 128 follows: 129

A between-receiver single-difference (BRSD) tightly cou-130 pled GNSS/INS integration mode is designed. This mode 131 reduces the effect of biases such as satellite-related error 132

and atmospheric error, and allows for full utilization of 133 more available GNSS observations. 134

Based on the tightly coupled model, a RANSAC-based 135 fault detection algorithm is presented. It can directly utilize 136 two satellites as subset sample with the help of inertial 137 navigation information. In addition, the global proportion 138 statistics method is introduced into the typical RANSAC 139 algorithm to further ensure detection reliability. 140

This article mainly presents the feasibility of RANSAC-based 141 algorithm to detecting faults in tightly coupled GNSS/INS inte-142 gration. The rest of this article is organized as follows. Section II 143 illustrates single-difference tightly coupled GNSS/INS integra-144 tion. Section III briefly introduces the principle of the RANSAC 145 algorithm. Section IV expounds on the RANSAC-based fault 146 detection and exclusion algorithm for single-difference tightly 147 coupled GNSS/INS integration. In Section V, the effect of the 148 main influencing factors on the proposed fault detection method 149 is analyzed and validated. In Section VI, land vehicle tests, 150 including typical scenarios, are conducted, and the experimental 151 results are analyzed and discussed. Finally, the conclusion and 152 characteristics of the proposed RANSAC-based fault detection 153 and exclusion algorithm are summarized in Section VII. 154

II. TIGHTLY COUPLED GNSS/INS INTEGRATED NAVIGATION 155

An observation model of tightly coupled GNSS/INS integra-156 tion can be constructed according to a GNSS positioning algo-157 rithm. Here, it is based on the between-receiver single-difference 158 model to avoid the consumption of observation information and 159 reduce the waste of effective parameters. 160

An augmented Kalman filter is applied to online estimate 161 and compensate for sensor errors, including IMU error, single-162 difference GNSS clock error and ambiguity. Fig. 1 shows a block 163 diagram of tightly coupled GNSS RTK/INS integration. Because 164 tightly coupled GNSS/INS integration research is relatively 165 mature, the design of the state model and observation model 166 is only briefly described. 167

A. State Model

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In GNSS/INS integration, the error state equations of the 169 Kalman filter are commonly based on the error dynamic equa-170 tions of the INS. The propagation of IMU errors in a given frame 171 can be defined by a set of coupled differential equations based 172 on the inertial navigation equations. Considering the IMU error, 173 the INS error dynamic equations with respect to the navigation 174 reference frame can be written as follows [25]: 175

168

$$egin{aligned} \delta \dot{m{r}}^n &= F \cdot \delta m{r}^n + \delta m{v}^n \ \delta \dot{m{v}}^n &= C_b^n \delta m{f}^b + C_b^n m{f}^b imes \phi - (2m{\omega}_{ie}^n + m{\omega}_{en}^n) imes \delta m{v}^n \ &+ m{v}^n imes (2\deltam{\omega}_{ie}^n + \deltam{\omega}_{en}^n) + \deltam{g}^n \ \dot{\phi} &= -m{\omega}_{in}^n imes \phi - C_b^n \deltam{\omega}_{ib}^b + \deltam{\omega}_{in}^n \ \dot{m{b}}_g &= -rac{1}{T}m{b}_g + m{w}_{bg} \ \dot{m{b}}_a &= -rac{1}{T}m{b}_a + m{w}_{ba} \end{aligned}$$

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Fig. 1. Block diagram of tightly coupled GNSS RTK/INS integration based on a between-receiver single-difference model.

$$\dot{s}_g = -\frac{1}{T}s_g + w_{sg}$$
$$\dot{s}_a = -\frac{1}{T}s_a + w_{sa}$$
(1)

176 where F is the coefficient matrix of the position error; δr^n , δv^n and ϕ represent the position, velocity and attitude error in 177 the navigation frame, respectively, and $\delta \dot{r}^n$, $\delta \dot{v}^n$ and $\dot{\phi}$ are the 178 corresponding time derivative; f^b is the specific force outputted 179 by the accelerometers; δf^b and $\delta \omega_{ib}^b$ represent the sensor errors 180 181 of the accelerometers and gyroscopes, including the bias (b_q and b_a) and scale factor (s_g and s_a) which are modeled as 1st 182 Gauss-Markov process (where T is the correlation time and w is 183 the driven white noise) and augmented to the error state vector 184 for online estimation and compensation; C_b^n is the direction 185 cosine matrix from the IMU frame to the navigation frame; ω_{en}^{n} , 186 ω_{ie}^{n} and ω_{in}^{n} represent the angular rates of the navigation frame 187 relative to the Earth frame, the Earth frame relative to the inertial 188 frame and the navigation frame relative to the inertial frame in 189 the navigation frame, respectively, and $\delta \omega_{en}^n$, $\delta \omega_{ie}^n$ and $\delta \omega_{in}^n$ are 190 191 the corresponding angular rate errors; δq^n is the normal gravity error at the local position; the superscripts n and b represent 192 the navigation frame and the IMU frame, respectively; and \times 193 represents the cross product of vectors. 194

A between-receiver single-difference model can reduce the 195 effect of satellite-related errors (e.g., clock error and orbit error) 196 and spatial propagation errors (e.g., ionosphere error and tropo-197 sphere error) with a baseline up to approximately 10 km [26]. 198 Compared to a double-difference model, a between-receiver 199 single-difference model needs to estimate the receiver clock 200 error. In this article, the GNSS clock model consists of two 201 parameters: clock error a_0 and clock drift a_1 , and the drift is 202 modeled as random walk. Hence, the GNSS clock model can be 203 written as 204

$$\dot{a}_0 = a_1 + w_0$$
$$\dot{a}_1 = w_1 \tag{2}$$

where w_0 is the white noise of the clock error and w_1 is the driven white noise of the random walk. The single-difference ambiguity ΔN is modeled as a random 207 constant, and the corresponding model can be expressed as 208

$$\Delta N_i = 0 \ (i = 1, \dots, m) \tag{3}$$

where m represents the number of single-difference carrier phase observations and i is a visible satellite for the rover and base station in the same epoch. 211

The tightly coupled GNSS/INS integration state model based on the between-receiver single-difference model can be formed by combining (1), (2) and (3).

B. Observation Model 215

GNSS observations consist of the pseudorange, carrier phase 216 and Doppler, and the corresponding between-receiver singledifference observation equations can be written as 218

$$P_{br}^{s} = P_{r}^{s} - P_{b}^{s} = \rho_{br}^{s} + T_{bias}^{sys} + \varepsilon$$
$$\tilde{\varphi}_{br}^{s} = \tilde{\varphi}_{r}^{s} - \tilde{\varphi}_{b}^{s} = \frac{1}{\lambda}\rho_{br}^{s} + \frac{1}{\lambda}T_{bias}^{sys} + \Delta N + \varepsilon$$
$$\tilde{D}_{br}^{s} = -\frac{1}{\lambda}\left[e_{r}^{s}\left(v^{s} - v_{r}\right) - e_{b}^{s}\left(v^{s} - v_{b}\right)\right] + T_{drift} + \varepsilon \quad (4)$$

where \tilde{P} , $\tilde{\varphi}$ and \tilde{D} are the pseudorange, carrier phase and 219 Doppler observations, respectively; the subscripts r and b rep-220 resent the rover and base station, respectively; ρ_{br}^{s} is the single-221 difference range; T_{bias}^{sys} is the single-difference clock error, and 222 it is the same as a_0 in (2); the superscript s represents a satellite; 223 $T_{drift} = (df_r - df_b)$, and it is the single-difference clock drift 224 that is the same as a_1 in (2); λ is the carrier wavelength; e_r^s 225 and e_b^s are the LOS unit vectors between the rover/base station 226 and the satellite, respectively; v^s , v_r and v_b are the velocities 227 of the satellite, rover and base station, respectively; and ε is the 228 observation error. 229

Here, the expression of the observations derived from inertial navigation is directly given below. The derived range and Doppler observations based on the between-receiver singledifference model can be written as

$$\hat{\rho}_{br}^{s} = \rho_{br}^{s} - \boldsymbol{e}_{r}^{s} \delta \boldsymbol{r}^{n} - \boldsymbol{e}_{r}^{s} \left[\left(C_{b}^{n} \boldsymbol{l}_{GNSS}^{b} \right) \times \right] \boldsymbol{\phi}$$

$$\hat{D}_{br}^{s} = -\frac{1}{\lambda} \left[\boldsymbol{e}_{r}^{s} \left(\boldsymbol{v}^{s} - \boldsymbol{v}_{r}^{n} \right) - \boldsymbol{e}_{b}^{s} \left(\boldsymbol{v}^{s} - \boldsymbol{v}_{b}^{n} \right) \right] + \frac{1}{\lambda} \boldsymbol{e}_{r}^{s} \delta \boldsymbol{v}^{n} \quad (5)$$

where l_{GNSS}^{b} represents the lever arm between the GNSS antenna and IMU center.

Combining (4) and (5) yields the observation equation of tightly coupled GNSS/INS integration based on the betweenreceiver single-difference model as follows:

$$z_{P} = -\boldsymbol{e}_{r}^{s}\delta\boldsymbol{r}^{n} - \boldsymbol{e}_{r}^{n}\left[\left(C_{b}^{n}\boldsymbol{l}_{GNSS}^{b}\right)\times\right]\phi - T_{bias}^{sys} + \varepsilon$$

$$z_{\phi} = -\frac{1}{\lambda}\boldsymbol{e}_{r}^{s}\delta\boldsymbol{r}^{n} - \frac{1}{\lambda}\boldsymbol{e}_{r}^{s}\left[\left(C_{b}^{n}\boldsymbol{l}_{GNSS}^{b}\right)\times\right]\phi$$

$$-\frac{1}{\lambda}T_{bias}^{sys} - \Delta N + \varepsilon$$

$$z_{D} = \frac{1}{\lambda}\boldsymbol{e}_{r}^{s}\left\{\delta\boldsymbol{v}^{n} - \left[C_{\omega}\left(C_{b}^{n}\boldsymbol{l}_{GNSS}^{b}\times\right) + \left(C_{l}\boldsymbol{\omega}_{ib}^{b}\times\right)\right]\phi - C_{l}\boldsymbol{b}_{g}$$

$$-C_{l}diag\left(\boldsymbol{\omega}_{ib}^{b}\right)\boldsymbol{s}_{g}\right\} - T_{drift} + \varepsilon \qquad (6)$$

239 where

$$C_{l} = C_{b}^{m} \left(l_{GNSS}^{b} \times \right)$$
$$C_{\omega} = \left(\omega_{ie}^{n} \times \right) + \left(\omega_{en}^{n} \times \right)$$
(7)

240 III. RANSAC-Based Fault Detection and Exclusion for 241 GNSS/INS INTEGRATION

This section gives a brief introduction to the principle of conventional RANSAC algorithm, and then details in its application and improvement in the tightly coupled GNSS/INS integration.

245 A. Principle of RANSAC

The RANSAC algorithm utilizes a voting scheme to obtain 246 the optimal model. The implementation of this voting scheme 247 is based on two assumptions: the noisy features will not vote 248 consistently for any single model, and there are sufficient good 249 features. The basic RANSAC algorithm is fundamentally com-250 posed of iterative subset sampling and consistency checking 251 [15]. First, a sample subset containing minimal necessary data is 252 253 randomly selected, and the corresponding model parameters are calculated based on this sample subset. Second, a consistency 254 check is used to distinguish inliers consistent with the model 255 and outliers inconsistent with the model, and the correctness of 256 the model based on the first sample subset is evaluated by the 257 number of inliers. These two steps are iteratively repeated until 258 the model has the highest level of consistency (that is, the highest 259 number of inliers). 260

For a RANSAC algorithm, there are three main parameters: 261 the sample number of the subset, the inlier judgment threshold 262 and the maximum iteration [15]. The sample number of the sub-263 set depends on the minimum number of data elements required 264 265 for model estimation. The inlier judgment threshold is generally set according to the desired confidence level. RANSAC is a 266 nondeterministic algorithm in the sense that it produces a reason-267 able result only with a certain probability, with this probability 268 increasing as more iterations are allowed. However, iterating 269 through all subsets is too time-consuming for a large sample, 270

so it is necessary to set an iteration threshold to improve the 271 algorithm efficiency. 272

B. Subset Selection in TC-GNSS/INS Solution 273

The number of subset samples is the minimum number of 274 data elements required for model estimation, and it refers to 275 the minimum number of satellites for GNSS positioning in 276 tightly coupled GNSS RTK/INS integration. In the conventional 277 GNSS positioning solution, it is generally believed that at least 278 4 satellites are required to estimate three-dimensional position 279 and the receiver clock error. 280

Compared with the conventional GNSS solution, tightly 281 coupled integration increases the INS assistance; therefore, 4 282 satellites are not necessary. We have previously analyzed the 283 auxiliary effect of different numbers of satellites on the tightly 284 coupled integration, and it will not be repeated in this article. Our 285 preliminary work based on multiple field tests results show that 286 2 satellites with good geometric distributions can improve the 287 integrated navigation accuracy. Therefore, the number of subset 288 samples is 2 satellites in this article. This is also the advantage 289 of the proposed method over the conventional GNSS solution. 290

C. Inlier Judgment in TC-GNSS/INS Solution

The inlier judgment is based on whether the observed GNSS 292 range information is consistent with the model formed by the 293 current subset. Here, the integrated navigation results, which are 294 obtained from the tightly coupled integration solution assisted 295 by the 2 satellites in the subset, can be used to perform inverse 296 computation of the range observation. The derived range and 297 the real observed range outside the subset are used to construct 298 the range residual that is the basis of the inlier judgment. The 299 following analysis will illustrate the calculation process of the 300 range residual and its standard deviation with the carrier phase 301 observation as an example. 302

The between-receiver single-difference carrier phase observation $\tilde{\varphi}_{br}^{s}$ is given in (4); here, it is rewritten as 304

$$\tilde{\varphi}_{br}^{s} = \frac{1}{\lambda}\rho_{br}^{s} + \frac{1}{\lambda}T_{bias}^{sys} + \Delta N \tag{8}$$

291

The derived single-difference carrier phase $\hat{\varphi}^s_{br}$ can be expressed by 305

$$\hat{\varphi}_{br}^{s} = \frac{1}{\lambda}\hat{\rho}_{br}^{s} + \frac{1}{\lambda}\hat{T}_{bias}^{sys} + \Delta\hat{N}$$
(9)

where $\hat{\rho}_{br}^{s}$ and \hat{T}_{bias}^{sys} can be obtained from the model parameters. 307 However, $\Delta \hat{N}$ is unknown, because the estimated results based 308 on the subset only include the single-difference ambiguity of 309 the selected 2 satellites, and the ambiguity of the remaining 310 satellites outside the subset is presently unknown. Therefore, it 311 is necessary to eliminate the single-difference ambiguity. 312

In general, the ambiguity remains the same for two adjacent 313 epochs, so it can be removed using the between-epoch difference 314 to yield the following expression. 315

$$\nabla\Delta\tilde{\varphi}^{s} = \tilde{\varphi}^{s}_{br}\left(t_{2}\right) - \tilde{\varphi}^{s}_{br}\left(t_{1}\right) \tag{10}$$

where $\nabla \Delta \tilde{\varphi}^s$ is the double-difference range observation, and the double difference is a between-epoch single difference of a between-receiver single difference; t_1 and t_2 are two adjacent epochs.

According to (9), the derived double-difference carrier phase $\nabla \Delta \hat{\varphi}^s$ can be expressed by

$$\nabla\Delta\hat{\varphi}^{s} = \frac{1}{\lambda} \left(\hat{\rho}_{br}^{s}(t_{2}) + \hat{T}_{bias}^{sys}(t_{2}) \right) - \frac{1}{\lambda} \left(\hat{\rho}_{br}^{s}(t_{1}) + \hat{T}_{bias}^{sys}(t_{1}) \right)$$
(11)

Combining (10) and (11) yields the double-difference carrier phase residual as

$$\delta\varphi^s = \nabla\Delta\hat{\varphi}^s - \nabla\Delta\hat{\varphi}^s \tag{12}$$

The double-difference carrier phase residual is the basic parameter used for the inlier judgment, and the corresponding variance σ^2 (σ is the corresponding standard deviation) can be written as,

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 \tag{13}$$

where σ_1^2 represents the variance of the derived doubledifference carrier phase $\nabla \Delta \hat{\varphi}^s$ and σ_2^2 represents the variance of the observed double-difference carrier phase $\nabla \Delta \hat{\varphi}^s$.

According to (11), the variance of the derived doubledifference carrier phase can be expressed by

$$\sigma_1^2 = \frac{1}{\lambda^2} \left(\sigma_{\rho 2}^2 + \sigma_{t2}^2 + \sigma_{\rho 1}^2 + \sigma_{t1}^2 \right)$$
(14)

where $\sigma_{\rho 1}^2$ and $\sigma_{\rho 2}^2$ are the variances of the range derived from the INS at the adjacent two epochs; σ_{t1}^2 and σ_{t2}^2 are the variances of the single-difference clock errors at the adjacent two epochs, respectively, and they can be obtained from the state variance matrix of the Kalman filter.

Through linearization expansion and spatial transformation, and ignoring the small effect of the covariance $D_{r\phi}$ between position and attitude, the variance σ_{ρ}^2 of the range derived from the INS can be written as

$$\sigma_{\rho}^{2} = H_{r}C_{n}^{e}\left(D_{r,IMU}^{n} + H_{\phi}D_{\phi}H_{\phi}^{\mathrm{T}}\right)C_{n}^{e\,\mathrm{T}}H_{r}^{\mathrm{T}} \qquad (15)$$

where H_r is the linearized matrix of the single-difference range; C_n^e is the direction cosine matrix from the navigation frame to the Earth frame; H_{ϕ} is the designed attitude matrix; $D_{r,IMU}^n$ is the variance matrix of the INS position error; and D_{ϕ} is the variance matrix of the INS attitude error.

According to (4) and (10), σ_2^2 can be calculated by the following equation.

$$\sigma_2^2 = n_{r2} + n_{b2} + n_{r1} + n_{b1} \tag{16}$$

where n_{r1} and n_{r2} are the variances of the measurement noise of the rover station at the two adjacent epochs; n_{b1} and n_{b2} are the variances of the measurement noise of the base station at the two adjacent epochs.

Theoretically, the constructed double-difference carrier phase residual is normally distributed, so the criterion of the inlier and the outlier can be written as,

$$\begin{cases} \delta \varphi^s < T1, \quad s \in inlier\\ \delta \varphi^s \ge T1, \quad s \in outlier \end{cases}$$
(17)

where the threshold value (denoted by T1 in Section III), which depends on the level of the required confidence, can be set as a multiple of the standard deviation of the residual. For example, the confidence level is 99.73% when the threshold is set to 3σ . 359

D. Subset Iteration in TC-GNSS/INS Solution

Different from the computer vision, the maximum number 361 of iterations need not be limited when the RANSAC algorithm 362 is applied to fault detection and exclusion of tightly coupled 363 integration. This is because the number of visible satellites is 364 limited, resulting in a small subset size with only 2 satellites. 365 However, subset construction requires consideration of the geo-366 metric distribution of these 2 satellites to ensure the accuracy of 367 the tightly coupled integration. From experience, it is better that 368 the azimuth difference between the 2 selected satellites generally 369 ranges from 60° to 120°. 370

On the basis of the conventional RANSAC algorithm as 371 described in Section III, the proposed algorithm adds the global 372 proportion statistics of faults to ensure detection reliability. 373 The global proportion statistics of faults consist of two steps: 374 recording the number of satellites classified as faults during 375 subset iteration and calculating the percentage of faults for each 376 satellite. Note that when satellites are included in the subset, 377 they are not classified as faults, and there are differences in the 378 number of satellites involved in constructing subsets. To exclude 379 the influence of these factors, the global proportion statistics of 380 the faults can be expressed as 381

$$R_a = \frac{OC}{SN - ISN} \tag{18}$$

where R_a represents the ratio of faults; OC is the number of 382 satellites classified as faults; SN is the total number of subsets 383 with 2 satellites in the current epoch; and ISN is the number of 384 detected satellites involved in the subset. 385

$$\begin{cases} R_a < T3, & s \in fault - free \\ R_a \ge T3, & s \in fault \end{cases}$$
(19)

In Section IV-B, we will delve into the impact of threshold 391 T3 value on fault detection. It's important to note that unlike 392 T1, which can be stochastically related to the probability of 393 false positives, T3 is determined by balancing the recall and 394 precision of fault detection. 395

E. Algorithm Framework of RANSAC-Based FDE in TC 396 GNSS/INS Solution 397

Fig. 2 shows the flow of the RANSAC-based fault detection398and exclusion of the tightly coupled integration. Block ① on399the left shows the operations performed for each subset. First,400tightly coupled integration based on the 2 selected satellites is401conducted to obtain the integrated solution. Second, the double-402difference residual and the corresponding standard deviation of403



Fig. 2. Flow chart of RANSAC-based fault detection and exclusion of tightly coupled GNSS/INS integration.

satellites outside the subset are calculated. Finally, the residual of each satellite is compared with the preset threshold T1, and the number of satellites classified as faults is recorded. Here, threshold T1, which is not constant, is related to the standard deviation of the double-difference residual.

The algorithm flow continues to the proposed global fault 409 proportion statistics procedure shown in block (2) on the right 410 when all subsets have been iterated and processed. The ratio 411 412 of satellites classified as faults is calculated and compared with preset threshold T3 for reliable fault detection and exclusion. 413 In general, the smaller the threshold is, the easier it is to detect 414 satellite observation faults, but the possibility of false positives is 415 also higher. Conversely, the larger the threshold is, the more dif-416 ficult it is to detect faults, but the possibility of false positives is 417 also lower. Hence, a reasonable threshold is a key parameter 418 to ensure the effectiveness of fault detection. The effect of the 419 threshold on the detection performance will be analyzed in the 420 following section. 421

422 IV. EFFECT ANALYSIS OF PARAMETERS ON RANSAC-BASED 423 FAULT DETECTION PERFORMANCE

In this section, we predominantly examine the influence of two parameters on the effectiveness of the proposed RANSACbased fault detection algorithm: the threshold T3 and the quantity of faulty satellites. Our goal is to provide a quantitative analysis of the algorithm's performance. To achieve this, we

TABLE I Optimized Specifications of ICM20602

Sensor	Parameters	ICM20602
Gyro	In-run bias instability (°/hr) White noise (°/ \sqrt{hr})	50 0.24
Accel.	In-run bias instability (ug) White noise (m/s/ √ hr)	250 0.24

introduced artificial cycle slips (an example of step errors) with 429 varying magnitudes into the raw carrier phase observations 430 collected from a vehicle-mounted rover receiver in an open-sky 431 environment. The magnitudes are in order as follows: 0.5 cycles, 432 1.0 cycle, 2.0 cycles and 3.0 cycles, denoted as 0.5c, 1c, 2c, and 433 3c, respectively. The number of visible satellites was limited to 434 12. A low-end MEMS grade GNSS/INS system with ICM20602 435 from TDK InvenSense was used for processing and analysis. 436 Table I lists the optimized specifications of the MEMS IMU. 437

A. Performance Evaluation Metrics

For the statistical classification problem, a confusion matrix is a specific table layout that allows visualization of the performance of an algorithm [27]. For binary classification, the scheme of the confusion matrix is shown in Table II. Each row of the matrix represents the instances in an actual class, while each column represents the instances in a predicted class. The

	TABLE II
CONFUSIO	N MATRIX FOR BINARY CLASSIFICATION

Total		Predicted class		
		Positive	Negative	
Actual	Positive	True Positive (TP)	False Negative (FN)	
class	Negative	False Positive (FP)	True Negative (TN)	

confusing matrix can make it easy to see whether the system isconfusing two classes.

447 The confusion matrix for binary classification shown in Table II presents four classification results: "TP" is the true 448 positive value, which is the number of positive observations 449 classified correctly; "TN" is the true negative value, which is 450 the number of negative observations classified correctly; "FP" 451 452 is the false positive value, which is the number of actual negative observations classified as positive; and "FN" is the false nega-453 tive value, which is the number of actual positive observations 454 classified as negative. 455

In essence, fault detection is a binary classification problem,
so the performance evaluation metrics of the fault detection
algorithm were borrowed from the terminology and derivations
of a confusion matrix [27]. The calculations of the performance
metrics, including the accuracy (ACC), precision (PRE), recall
(REC) and F-score (Fs) values, are made according to (20)–(23).

$$ACC = \frac{TP + TN}{TP + TN + FP + FN}$$
(20)

$$PRE = \frac{TP}{FP + TP} \tag{21}$$

$$REC = \frac{TP}{FN + TP}$$
(22)

$$Fs = 2 \times \frac{PRE \times REC}{PRE + REC}$$
(23)

It should be noted that the ACC reflects the probability of 462 observations classified correctly, but it can be misleading if 463 used with imbalanced datasets. The PRE represents the ratio 464 of the detected actual negative observations relative to those 465 classified as negative observations, and the lower the PRE is, 466 the higher the false detection rate. The REC represents the ratio 467 of the detected actual negative observations relative to all actual 468 negative observations, and the lower the REC is, the higher the 469 missed detection rate. The Fs value is the harmonic mean of the 470 PRE and REC. 471

In fault detection, missed detection can lead to faults being 472 included in the integrated navigation solution and producing 473 incorrect results, while false detection can result in accurate 474 observations not being used to reduce the integrated navigation 475 accuracy. Therefore, while guaranteeing a certain REC level, the 476 PRE magnitude should be considered. In the following, we will 477 utilize these two metrics to analyze the effect of parameters on 478 detection performance, and the analysis results are displayed in 479 the form of a percentage of performance metrics. 480



Fig. 3. Performance metric curves representing the effect of threshold T3 on RANSAC-based fault detection with different numbers of artificial cycle slips.

B. Effect of Thresholds on Detection Performance

Fig. 3 shows the REC and PRE representing the effect of 489 threshold T3 on RANSAC-based fault detection with different 490 artificial cycle slips. There are 6 satellites with faults, and 491 threshold T3 varies from 0.2 to 0.8. Considering the REC, the 492 value with 0.5 cycle slips is the lowest under the same threshold 493 T3, which indicates that the detection of 0.5 cycle slips is the 494 most difficult. The REC values of all cycle slips decrease as 495 threshold T3 increases, which indicates that the missed detection 496 rate increases as threshold T3 increases. 497

Considering the PRE, there is less variation in the value with 498 0.5 cycle slips when threshold T3 is changed, and the PRE value 499 can be basically controlled above 80%. The PRE value of $1 \sim 3$ 500 cycle slips is less than 60% when threshold T3 is less than 0.5 to 501 increase the false detection rate. If a small cycle slip (e.g., less 502 than (0.5c) is the main error, the threshold T3 can be set to (0.4). 503 If the large cycle slip (e.g., larger than 1.0c) is the main error, 504 the threshold T3 should be set to 0.7. The threshold T3 can be 505 set to 0.6 when taking into account cycle slips of $0.5c \sim 3.0c$. 506

C. Effect of Fault Number on Detection Performance

Fig. 4 shows the performance metric curves representing the 508 effect of the number of faulty satellites on the RANSAC-based fault detection with different artificial cycle slips. The total number of visible satellites is 12, and the number of satellites 511 with artificial faults is $1 \sim 8$. The REC value of the detection 512

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Fig. 4. Performance metrics curves representing the effect of the number of faulty satellites on the RANSAC-based fault detection with different numbers of artificial cycle slips.

algorithm decreases as the number of faulty satellites increases,
especially for 0.5 cycle slips. The REC value with 0.5 cycle
slips basically remains above 90% when the number of faulty
satellites is less than 4.

Different from the REC value, the PRE value does not al-517 ways decrease as the number of faulty satellites increases. The 518 fluctuation of the PRE curve with 0.5 cycle slips is small, and 519 the overall performance decreases with an increasing number 520 of faulty satellites, while the PRE curves with $1 \sim 3$ cycle slips 521 show a trend of first decreasing and then increasing. The larger 522 the threshold T3 is, the greater the number of faulty satellites at 523 the minimum value of the curve. The number of faulty satellites 524 corresponding to the minimum value of the curve is 4 and 6 525 when the threshold T3 is 0.4 and 0.6, respectively; the number 526 of faulty satellites is 8 when T3 is 0.7, which makes the curve 527 show a monotonically decreasing trend. 528

For the special trends in the PRE curve, since the total number of satellites is fixed, an increase in the number of faulty satellites results in a decrease in the number of normal satellites. At this time, the detection algorithm has the possibility of false detection, but the number of satellites that can be classified decreases, so the PRE value increases instead.

535

V. TESTS AND RESULTS

This section presents an analysis of positioning performance in typical urban scenarios and provides statistics from multiple tests conducted in urban environments. Section A focuses on navigation performance in various scenarios, while Section B



Fig. 5. Land vehicle test trajectory segmented with letters (To the left is north, generated by google earth).

TABLE III Scenario Descriptions of Different Road Segments

Road segment	Scenario description	Time Proportion
AB	Crossing an urban canyon, GNSS signals were blocked by buildings, and the number of visible satellites was approximately 6.	29.7%
BC	Under a viaduct, the number of visible satellites is less than 6.	15.8%
CD	On a viaduct, and GNSS signals were blocked for 49 seconds at the end of the viaduct due to the noise barrier.	18.2%
DE	Tunnel, there is no GNSS signal.	24.2%
EF	Boulevard, the number of visible satellites is more than 6.	12.1%

discusses the effectiveness and availability of the proposed 540 RANSAC-based fault detection and exclusion method. 541

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A. Performance Analysis of Typical Urban Scenarios

To further explore the comprehensive performance of the 543 RANSAC-based fault detection of tightly coupled integration in 544 typical urban scenarios, a land vehicle test covering buildings, 545 tunnels, and viaducts was conducted in Wuhan city. Fig. 5 shows 546 the test trajectory, and the trajectory distance is approximately 547 4.5 km. The detailed scenario descriptions of different road 548 segments marked with letters are listed in Table III, and the 549 vehicle speed is low in the downtown such as segment AB and 550 BC. 551

Fig. 6 shows the installation of the equipment used for the field 552 land vehicle test. The INSProbe is a MEMS grade GNSS/INS 553 integrated system with ICM20602 from TDK InvenSense, and 554 a NovAtel OEM718D card is used for GNSS data acquisition. 555 The POS620 is a navigation grade GNSS/INS integrated system 556 with a high-grade fiber optic gyro (FOG), and its postprocessing 557 smoothed results serve as the reference truth for data analysis. 558 The specifications of these two IMUs are listed in Table IV. 559

Various data processing modes are employed to evaluate the viability of the proposed fault detection method in urban areas. For a detailed description of the data processing mode, see Table V, which outlines the implementation of an innovationbased fault detection method utilizing the tightly coupled GNSS/INS integration. 560 561 562 563 564 565



Fig. 6. Installation of the equipment used for the field land vehicle test.

TABLE IV SPECIFICATIONS OF IMUS

Sensor	Parameters	ICM20602	POS620 (reference)
Gyro	In-run bias instability (°/hr) White noise (°/ \sqrt{hr})	50 0.24	0.03
Accel.	In-run bias instability (ug) White noise (m/s/ \sqrt{hr})	250 0.24	15 0.03

TABLE V DATA PROCESSING MODE DESCRIPTIONS

F	Process Mode	Mode Abbreviation	Description
	GNSS RTK	RTK	Forward processing of single frequency GPS/BDS data
gration	Loosely coupled	LC	Forward filter of RTK/INS loosely coupled integration with robust estimation based on innovation
GNSS/INS integ	Tightly coupled	TC1	Forward filter of RTK/INS tightly coupled integration with robust estimation based on innovation
	Tightly coupled with RANSAC	TC2	Forward filter of RTK/INS tightly coupled integration with RANSAC-based fault detection

Fig. 7 shows the position error of the different processing 566 modes, and the number of satellites, including visible satellites, 567 satellites with cycle slip, and satellites rejected. The GNSS 568 interruption interval is marked on the horizontal axis with a 569 yellow block. Overall, the TC2 mode boasts good position 570 accuracy, particularly in challenging situations, and is supported 571 by the proposed RANSAC-based method for fault detection. 572 The positioning performance is analyzed segment by segment 573 to show the characteristics of different processing modes in 574 different scenarios. 575

Before segment AB, the RTK mode can maintain a fixed solution. During segment AB, the position accuracy and continuity
of the RTK mode are significantly reduced as the number of
satellites gradually decreases, and the position accuracy of the
LC mode is affected by the GNSS positioning performance.

TABLE VI PERFORMANCE EVALUATION METRICS DESCRIPTIONS

Metric	Metric Descriptions	
Max	Maximum of the absolute value of the navigation error.	
RMS	Root mean square of the navigation error.	
CDF95	Error value corresponding to the cumulative distribution function with 95%.	
Fixed rate	Proportion of epochs with ambiguity correctly fixed.	
Valid rate	Proportion of epochs with position error is less than 5.0 m.	
Success rate	Proportion of epochs with correct positioning.	

For the TC1 and TC2 modes, there is no obvious difference in 581 cycle slip detection and satellite rejection, and the corresponding 582 position accuracy can be controlled within 2.0 m even when 583 there are fewer than 4 satellites. During segment BC, the number 584 of visible satellites is approximately $3 \sim 4$, which is caused by 585 severe GNSS signal occlusion caused by the viaduct. Although 586 the position accuracy of all modes is poor, that of the TC1 and 587 TC2 modes can be controlled within 5.0 m and has a relatively 588 good position accuracy compared with the RTK and LC modes. 589

During segment CD, there is a difference in cycle slip detec-590 tion and satellite rejection for the TC1 and TC2 modes, and the 591 RANSAC-based fault detection method guarantees the tightly 592 coupled integrated position accuracy of the TC2 mode in the 593 challenging scenario. The correct fault detection of the TC2 594 mode before entering the tunnel reduces the position error diver-595 gence level compared with the TC1 mode. The GNSS signals 596 of segment DE are interrupted for approximately 3 minutes, 597 and the horizontal position error of the TC2 mode diverges to 598 approximately 10 m, while the horizontal position error of the 599 TC1 mode reaches 30 m. 600

During segment EF, a large number of fault-free satellites 601 were mistakenly eliminated in the TC1 mode, and a long time 602 was required to achieve the convergence of position error. Conversely, the TC2 mode completed the rapid convergence of 604 position error because of the RANSAC-based fault detection 605 method, which effectively controlled the false detection rate and 606 the missed detection rate. 607

In a typical environment, the RTK and LC modes can experi-608 ence significant disruption to their positioning performance from 609 external environmental disturbances. However, the TC mode 610 has the capability to leverage the raw GNSS observations to 611 achieve a reliable GNSS/INS integration solution even when the 612 number of satellites is less than four. Notably, the TC2 mode has 613 implemented a RANSAC-based fault detection mechanism to 614 further enhance positioning accuracy in challenging scenarios. 615

In addition, we also used statistical results for performance 616 evaluation, and the performance evaluation metrics are defined 617 as shown in Table VI. 618

Fig. 8 shows the performance evaluation metrics of the dif-619 ferent processing modes. The position accuracy represented by 620 the Max, RMS and CDF95 of the TC2 mode is significantly 621 better than that of the TC1 and LC modes. Since faults are not 622 correctly detected and eliminated before and after the tunnel, 623 the north position error of the TC1 mode is larger than that 624 of the LC mode. The success rate of the RTK mode is less 625 than 50% because there is frequent GNSS signal interruption 626



Fig. 7. Position error of the different processing mode and the number of satellites.



Fig. 8. Performance evaluation metrics of the different processing modes.

caused by the external environment. Although the LC mode can
maintain continuous positioning, the corresponding valid rate is
only 33%. The valid rate of the two tightly coupled modes is
more than 60%, and compared with the TC1 mode, the valid
rate and fixed rate of the TC2 mode are increased by 29% and
19%, respectively.



Fig. 9. Performance statistics of multiple tests in urban environment.

633 B. Performance Statistics of Multiple Urban Environments

Multiple land vehicle tests were conducted in a complex urban environment to evaluate the feasibility of the RANSAC-based fault detection in tightly coupled integration. Here, the total time length of field test is approximately 7 hours and the environmental conditions include the downtown, campus, city tunnel and viaduct etc. Fig. 9 presents the statistics obtained from these tests. Overall, the maximum position errors and the CDF95 values of the TC2 mode are smaller than those of the

TC1 mode, and the fixed rate and valid rate are significantly 642 higher than those of the TC1 mode. The proposed RANSAC-643 based fault detection algorithm significantly improved the north 644 645 and east position accuracy (in terms of CDF95) of the tightly coupled mode in the comprehensive scenario, with an average 646 increase of 45% and 42% respectively. This indicates that the 647 positioning performance of the TC2 mode has been enhanced by 648 the RANSAC-based fault detection algorithm in complex urban 649 environments. 650

651 However, in relation to data 2, the TC2 mode displays smaller maximum position errors and CDF95 values compared to the 652 TC1 mode, yet its fixed rate remains lower. This discrepancy 653 suggests that the proposed fault detection algorithm has yielded 654 a high false positive rate, incorrectly classifying normal GNSS 655 observations as faults. The reason behind the unsatisfactory PRE 656 value can be attributed to the greater emphasis given to the 657 REC value for ensuring position error level. This also highlights 658 the flaws in the threshold setting approach of the proposed 659 algorithm. Fixed thresholds may not be suitable for all scenarios, 660 thereby rendering the algorithm inaccurate. 661

Based on the above analysis of land vehicle tests, it can be seen the TC2 mode can provide navigation information with high performance due to RANSAC-based fault detection and exclusion, and it is better that the thresholds should be adaptively adjusted to ensure the applicability of the proposed algorithm.

VI. CONCLUSION

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This work draws on the application of the RANSAC algorithm 669 for GNSS fault detection, and proposes a RANSAC-based fault 670 detection and exclusion of a tightly coupled GNSS RTK/INS 671 integration for a high-accuracy positioning solution in urban 672 environments. The between-receiver single-difference tightly 673 coupled mode was applied to fully utilize valid GNSS obser-674 vations. The characteristics of RANSAC-based algorithm for 675 tightly coupled integration were analyzed from the aspects of 676 subset selection, inlier judgment, subset iteration and so on. 677 A fault global proportion statistics was extended to the typical 678 RANSAC algorithm to enhance the detection reliability. 679

Simulation tests, where artificial cycle slips of different mag-680 nitudes were inserted into raw GNSS observations in an open-681 sky environment, were conducted to analyze the performance 682 of the proposed RANSAC-based fault detection algorithm. The 683 test results show that the proposed algorithm can effectively 684 detect small faults and multiple faults, and the detection rates 685 for 0.5c and $1c \sim 3c$ slips were approximately 70% and 90%, 686 respectively. Furthermore, land vehicle tests that included typi-687 cal scenarios in complex urban environments were conducted 688 to further investigate the comprehensive performance of the 689 proposed algorithm. The results indicate that the tightly coupled 690 mode was more suitable for changeable GNSS environments 691 compared to the loosely coupled mode; and with the help of the 692 proposed RANSAC-based fault detection algorithm, the north 693 and east position accuracy (in terms of CDF95) of the tightly 694 coupled mode in the comprehensive scenario was improved by 695 696 an average of 45% and 42%.

The proposed RANSAC-based fault detection algorithm can be further applied to multi-sensor information fusion, and guarantee a high level of accuracy and reliability in the positioning solution in harsh urban environments. Our subsequent work will thoroughly compare with the existing methods and optimize the threshold setting scheme to ensure the superiority and universality of the proposed algorithm. 703

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3

RANSAC-Based Fault Detection and Exclusion Algorithm for Single-Difference Tightly Coupled GNSS/INS Integration

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5 Abstract—There is an urgent need for high-accuracy and highreliability navigation and positioning in life safety fields such as 6 7 intelligent transportation and automotive driving, especially in complex urban environments. Although, compared with the GNSS 8 and loosely coupled integration, a tightly coupled GNSS/INS inte-9 gration can improve the positioning reliability by using raw obser-10 vations, it still suffers from external challenging environments such 11 as the multipath effect. Therefore, the fault detection algorithm is 12 a premise and guarantee to realize quality control of GNSS/INS 13 integration. Inspired by the application of the random sample con-14 sensus (RANSAC) algorithm in GNSS fault detection, this article 15 16 proposes a RANSAC-based fault detection and exclusion algorithm for single-difference tightly coupled GNSS/INS integration. Here, 17 18 a between-receiver single-difference (BRSD) model was designed to prevent the consumption of GNSS observations and reduce the 19 20 waste of effective parameters, and the global proportion statistics of faults were introduced into the typical RANSAC algorithm to 21 further ensure detection reliability. In this study, the effect of the 22 23 main parameters on the proposed detection algorithm was analyzed and verified by artificial cycle slips. Multiple filed tests, including 24 typical urban scenarios, were conducted to verify the feasibility 25 26 and effectiveness of the proposed method. The comprehensive 27 test results show that the north and east positioning accuracy in terms of cumulative distribution function (CDF, CDF = 95%) are 28 29 improved by 45% and 42% over the tightly coupled mode without the proposed detection method. 30

Index Terms—Fault detection, RANSAC, tightly coupled,
 between-receiver single difference, GNSS/INS integration.

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I. INTRODUCTION

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HE integration of the global navigation satellite system 34 (GNSS) with an inertial navigation system (INS) can 35 achieve complementary advantages, providing pose services 36 with high accuracy and continuity for the intelligent vehicle nav-37 igation and control. There has been an increasing demand for the 38 positioning accuracy and reliability of GNSS/INS integration, 39 especially using low-cost sensors (e.g., microelectromechanical 40 system (MEMS) inertial measurement unit (IMU)), in safety of 41 life applications such as intelligent driving [1], [2]. However, 42 complex urban environments bring severe challenges to GNSS 43 observation. For example, satellite visibility is completely or 44 partially obscured in urban environments, which results in a 45 decrease in GNSS positioning accuracy and continuity [3], [4]. 46

Tightly coupled (TC) GNSS/INS integration can directly 47 utilize raw GNSS observations for measurement updates and 48 performs better than loosely coupled (LC) integration in areas 49 with partially blocked GNSS access [5]. Although GNSS/INS 50 integration can ensure positioning continuity, satellite signals 51 are still interfered by the non-line-of-sight (NLOS) signals and 52 multipath effects, resulting in GNSS observation faults and 53 ultimately affecting the positioning accuracy and reliability 54 in challenging environments. Therefore, quality control is a 55 prerequisite to correctly detect faults and improve positioning 56 accuracy and reliability. Common GNSS/INS integration fault 57 detection methods are conducted by constructing test statistics 58 based on the innovation vector of a Kalman filter [6], [7]. These 59 methods apply quality control at the information fusion level 60 and are not effective for multiple faults detection. Classical 61 receiver autonomous integrity monitoring (RAIM) algorithms 62 have been developed to provide fault detection and exclusion 63 (FDE) [8], [9], but they generally work properly in the case of a 64 single fault and cannot provide reliable multiple faults detection 65 capabilities. Although there are some methods such as multiple 66 hypothesis solution separation (MHSS) and an advanced RAIM 67 (ARAIM) method to solving multiple faults, these methods will 68 be ineffective in presence of significantly large biases or large 69 proportion of faulty satellites [10], [11]. 70

Random sample consensus (RANSAC) can achieve correct71GNSS fault detection in cases of multiple and small faults, and72it is the research hotspot of GNSS fault detection and exclusion73[12]. RANSAC is an iterative method to estimate the parameters74of a mathematical model from a set of observed data that contains75

faults, and it can be interpreted as a fault detection method. The 76 RANSAC algorithm was first proposed by Fischler and Bolles 77 [13] and has been widely used in the field of computer vision 78 79 and is capable of interpreting or smoothing data containing a significant percentage of faults [14]. Schroth et al. [15] first 80 proposed the range consensus (RANCO) algorithm and the 81 suggestion range consensus (S-RANCO) algorithm to detect 82 faulty GNSS range measurements based on the elementary idea 83 of the RANSAC algorithm. Furthermore, Schroth et al. [16] 84 85 optimized the performance of RANCO by enhancing the subset evaluation, the subset selection algorithm and the modified 86 threshold definition to significantly reduce the missed detection 87 rate and false alarm rate. 88

On the basis of Schroth's research work, many performance 89 (in terms of accuracy, effectiveness, and stability) improve-90 ment methods have been studied. Groves and Jiang et al. [17], 91 [18] applied weighting based on consistency and C/N_0 to the 92 common RANSAC cost function to reduce the number of the 93 largest GNSS faults and used four GNSS measurements plus a 94 height-aiding measurement instead of 5 GNSS measurements 95 96 to improve the positioning accuracy. Su et al. [19] proposed a 97 fast RANSAC algorithm using geometric dilution of precision (GDOP), the line-of-sight (LOS) vector and singular value de-98 composition (SVD) for subset preselection to solve the large 99 100 computational load problem in the traditional RANSAC algorithm. An augmented version of the RANSAC algorithm that 101 performs a final range comparison using the state estimate ob-102 tained with only the inliers identified by RANSAC was proposed 103 for more reliable availability [20]. Zhao et al. [21] proposed a 104 modified RANCO algorithm based on a genetic algorithm to 105 106 inhibit the amount of exponential calculation. In addition, the RANSAC algorithm was introduced to protect the robustness 107 and accuracy of a multi-GNSS time-difference carrier phase 108 (TDCP) solution [22]. 109

Currently, the RANSAC algorithm is applied to the fault de-110 tection and exclusion of individual GNSS range measurements, 111 and the relevant research focuses on improving the compu-112 113 tational efficiency and fault identification precision. Although 114 some research has utilized the RANSAC algorithm to address the 115 issue of loosely coupled GNSS/INS integration as demonstrated in some studies [23], [24], a critical unresolved problem pertains 116 to the minimum number of satellites required in subset construc-117 tion. This issue remains unsolved, and it is still necessary to use 118 a minimum of four satellites. The existing relevant research does 119 not design the RANSAC-based algorithm in the tightly coupled 120 GNSS/INS integration, and not fully play the auxiliary role of 121 inertial navigation information in the subset construction. 122

Inspired by its application to GNSS positioning solu-123 tions, RANSAC is applied to single-difference tightly coupled 124 GNSS/INS integration for robust and high-accuracy positioning 125 in this study. The characteristics and contribution of RANSAC-126 127 based fault detection in the context of single-difference tightly coupled GNSS/INS integrated navigation can be summarized as 128 follows: 129

A between-receiver single-difference (BRSD) tightly cou-130 pled GNSS/INS integration mode is designed. This mode 131 reduces the effect of biases such as satellite-related error 132

and atmospheric error, and allows for full utilization of 133 more available GNSS observations. 134

Based on the tightly coupled model, a RANSAC-based 135 fault detection algorithm is presented. It can directly utilize 136 two satellites as subset sample with the help of inertial 137 navigation information. In addition, the global proportion 138 statistics method is introduced into the typical RANSAC 139 algorithm to further ensure detection reliability. 140

This article mainly presents the feasibility of RANSAC-based 141 algorithm to detecting faults in tightly coupled GNSS/INS inte-142 gration. The rest of this article is organized as follows. Section II 143 illustrates single-difference tightly coupled GNSS/INS integra-144 tion. Section III briefly introduces the principle of the RANSAC 145 algorithm. Section IV expounds on the RANSAC-based fault 146 detection and exclusion algorithm for single-difference tightly 147 coupled GNSS/INS integration. In Section V, the effect of the 148 main influencing factors on the proposed fault detection method 149 is analyzed and validated. In Section VI, land vehicle tests, 150 including typical scenarios, are conducted, and the experimental 151 results are analyzed and discussed. Finally, the conclusion and 152 characteristics of the proposed RANSAC-based fault detection 153 and exclusion algorithm are summarized in Section VII. 154

II. TIGHTLY COUPLED GNSS/INS INTEGRATED NAVIGATION 155

An observation model of tightly coupled GNSS/INS integra-156 tion can be constructed according to a GNSS positioning algo-157 rithm. Here, it is based on the between-receiver single-difference 158 model to avoid the consumption of observation information and 159 reduce the waste of effective parameters.

An augmented Kalman filter is applied to online estimate 161 and compensate for sensor errors, including IMU error, single-162 difference GNSS clock error and ambiguity. Fig. 1 shows a block 163 diagram of tightly coupled GNSS RTK/INS integration. Because 164 tightly coupled GNSS/INS integration research is relatively 165 mature, the design of the state model and observation model is only briefly described. 167

A. State Model

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In GNSS/INS integration, the error state equations of the 169 Kalman filter are commonly based on the error dynamic equa-170 tions of the INS. The propagation of IMU errors in a given frame 171 can be defined by a set of coupled differential equations based 172 on the inertial navigation equations. Considering the IMU error, 173 the INS error dynamic equations with respect to the navigation 174 reference frame can be written as follows [25]: 175

$$egin{aligned} \delta \dot{m{r}}^n &= F \cdot \delta m{r}^n + \delta m{v}^n \ \delta \dot{m{v}}^n &= C_b^n \delta m{f}^b + C_b^n m{f}^b imes \phi - (2m{\omega}_{ie}^n + m{\omega}_{en}^n) imes \delta m{v}^n \ &+ m{v}^n imes (2\deltam{\omega}_{ie}^n + \deltam{\omega}_{en}^n) + \deltam{g}^n \ \dot{\phi} &= -m{\omega}_{in}^n imes \phi - C_b^n \deltam{\omega}_{ib}^b + \deltam{\omega}_{in}^n \ \dot{m{b}}_g &= -rac{1}{T}m{b}_g + m{w}_{bg} \ \dot{m{b}}_a &= -rac{1}{T}m{b}_a + m{w}_{ba} \end{aligned}$$

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Fig. 1. Block diagram of tightly coupled GNSS RTK/INS integration based on a between-receiver single-difference model.

$$\dot{s}_g = -\frac{1}{T}s_g + w_{sg}$$
$$\dot{s}_a = -\frac{1}{T}s_a + w_{sa} \tag{1}$$

176 where F is the coefficient matrix of the position error; δr^n , δv^n and ϕ represent the position, velocity and attitude error in 177 the navigation frame, respectively, and $\delta \dot{r}^n$, $\delta \dot{v}^n$ and ϕ are the 178 corresponding time derivative; f^b is the specific force outputted 179 by the accelerometers; δf^b and $\delta \omega_{ib}^b$ represent the sensor errors 180 181 of the accelerometers and gyroscopes, including the bias (b_q and b_a) and scale factor (s_g and s_a) which are modeled as 1st 182 Gauss-Markov process (where T is the correlation time and w is 183 the driven white noise) and augmented to the error state vector 184 for online estimation and compensation; C_b^n is the direction 185 cosine matrix from the IMU frame to the navigation frame; ω_{en}^{n} , 186 ω_{ie}^n and ω_{in}^n represent the angular rates of the navigation frame 187 relative to the Earth frame, the Earth frame relative to the inertial 188 frame and the navigation frame relative to the inertial frame in 189 the navigation frame, respectively, and $\delta \omega_{en}^n$, $\delta \omega_{ie}^n$ and $\delta \omega_{in}^n$ are 190 the corresponding angular rate errors; δq^n is the normal gravity 191 error at the local position; the superscripts n and b represent 192 the navigation frame and the IMU frame, respectively; and \times 193 represents the cross product of vectors. 194

A between-receiver single-difference model can reduce the 195 effect of satellite-related errors (e.g., clock error and orbit error) 196 and spatial propagation errors (e.g., ionosphere error and tropo-197 sphere error) with a baseline up to approximately 10 km [26]. 198 Compared to a double-difference model, a between-receiver 199 single-difference model needs to estimate the receiver clock 200 error. In this article, the GNSS clock model consists of two 201 parameters: clock error a_0 and clock drift a_1 , and the drift is 202 modeled as random walk. Hence, the GNSS clock model can be 203 204 written as

$$\dot{a}_0 = a_1 + w_0$$
$$\dot{a}_1 = w_1 \tag{2}$$

where w_0 is the white noise of the clock error and w_1 is the driven white noise of the random walk. The single-difference ambiguity ΔN is modeled as a random 207 constant, and the corresponding model can be expressed as 208

$$\Delta N_i = 0 \ (i = 1, \dots, m) \tag{3}$$

where m represents the number of single-difference carrier 209 phase observations and i is a visible satellite for the rover and 210 base station in the same epoch. 211

The tightly coupled GNSS/INS integration state model based on the between-receiver single-difference model can be formed by combining (1), (2) and (3).

B. Observation Model 215

GNSS observations consist of the pseudorange, carrier phase 216 and Doppler, and the corresponding between-receiver singledifference observation equations can be written as 218

$$P_{br}^{s} = P_{r}^{s} - P_{b}^{s} = \rho_{br}^{s} + T_{bias}^{sys} + \varepsilon$$
$$\tilde{\varphi}_{br}^{s} = \tilde{\varphi}_{r}^{s} - \tilde{\varphi}_{b}^{s} = \frac{1}{\lambda}\rho_{br}^{s} + \frac{1}{\lambda}T_{bias}^{sys} + \Delta N + \varepsilon$$
$$\tilde{D}_{br}^{s} = -\frac{1}{\lambda}\left[\boldsymbol{e}_{r}^{s}\left(\boldsymbol{v}^{s} - \boldsymbol{v}_{r}\right) - \boldsymbol{e}_{b}^{s}\left(\boldsymbol{v}^{s} - \boldsymbol{v}_{b}\right)\right] + T_{drift} + \varepsilon \quad (4)$$

where \tilde{P} , $\tilde{\varphi}$ and \tilde{D} are the pseudorange, carrier phase and 219 Doppler observations, respectively; the subscripts r and b rep-220 resent the rover and base station, respectively; ρ_{br}^{s} is the single-221 difference range; T_{bias}^{sys} is the single-difference clock error, and 222 it is the same as a_0 in (2); the superscript s represents a satellite; 223 $T_{drift} = (df_r - df_b)$, and it is the single-difference clock drift 224 that is the same as a_1 in (2); λ is the carrier wavelength; e_r^s 225 and e_b^s are the LOS unit vectors between the rover/base station 226 and the satellite, respectively; v^s , v_r and v_b are the velocities 227 of the satellite, rover and base station, respectively; and ε is the 228 observation error. 229

Here, the expression of the observations derived from inertial navigation is directly given below. The derived range and Doppler observations based on the between-receiver singledifference model can be written as

$$\hat{\rho}_{br}^{s} = \rho_{br}^{s} - \boldsymbol{e}_{r}^{s} \delta \boldsymbol{r}^{n} - \boldsymbol{e}_{r}^{s} \left[\left(C_{b}^{n} \boldsymbol{l}_{GNSS}^{b} \right) \times \right] \boldsymbol{\phi}$$

$$\hat{D}_{br}^{s} = -\frac{1}{\lambda} \left[\boldsymbol{e}_{r}^{s} \left(\boldsymbol{v}^{s} - \boldsymbol{v}_{r}^{n} \right) - \boldsymbol{e}_{b}^{s} \left(\boldsymbol{v}^{s} - \boldsymbol{v}_{b}^{n} \right) \right] + \frac{1}{\lambda} \boldsymbol{e}_{r}^{s} \delta \boldsymbol{v}^{n} \quad (5)$$

where l_{GNSS}^{b} represents the lever arm between the GNSS antenna and IMU center.

Combining (4) and (5) yields the observation equation of tightly coupled GNSS/INS integration based on the betweenreceiver single-difference model as follows:

$$z_{P} = -\boldsymbol{e}_{r}^{s}\delta\boldsymbol{r}^{n} - \boldsymbol{e}_{r}^{n}\left[\left(C_{b}^{n}\boldsymbol{l}_{GNSS}^{b}\right)\times\right]\phi - T_{bias}^{sys} + \varepsilon$$

$$z_{\phi} = -\frac{1}{\lambda}\boldsymbol{e}_{r}^{s}\delta\boldsymbol{r}^{n} - \frac{1}{\lambda}\boldsymbol{e}_{r}^{s}\left[\left(C_{b}^{n}\boldsymbol{l}_{GNSS}^{b}\right)\times\right]\phi$$

$$-\frac{1}{\lambda}T_{bias}^{sys} - \Delta N + \varepsilon$$

$$z_{D} = \frac{1}{\lambda}\boldsymbol{e}_{r}^{s}\left\{\delta\boldsymbol{v}^{n} - \left[C_{\omega}\left(C_{b}^{n}\boldsymbol{l}_{GNSS}^{b}\times\right) + \left(C_{l}\boldsymbol{\omega}_{ib}^{b}\times\right)\right]\phi - C_{l}\boldsymbol{b}_{g}$$

$$-C_{l}diag\left(\boldsymbol{\omega}_{ib}^{b}\right)\boldsymbol{s}_{g}\right\} - T_{drift} + \varepsilon \qquad (6)$$

239 where

$$C_{l} = C_{b}^{n} \left(l_{GNSS}^{b} \times \right)$$
$$C_{\omega} = \left(\omega_{ie}^{n} \times \right) + \left(\omega_{en}^{n} \times \right)$$
(7)

240 III. RANSAC-Based Fault Detection and Exclusion for 241 GNSS/INS INTEGRATION

This section gives a brief introduction to the principle of conventional RANSAC algorithm, and then details in its application and improvement in the tightly coupled GNSS/INS integration.

245 A. Principle of RANSAC

The RANSAC algorithm utilizes a voting scheme to obtain 246 the optimal model. The implementation of this voting scheme 247 is based on two assumptions: the noisy features will not vote 248 consistently for any single model, and there are sufficient good 249 features. The basic RANSAC algorithm is fundamentally com-250 posed of iterative subset sampling and consistency checking 251 252 [15]. First, a sample subset containing minimal necessary data is 253 randomly selected, and the corresponding model parameters are calculated based on this sample subset. Second, a consistency 254 check is used to distinguish inliers consistent with the model 255 and outliers inconsistent with the model, and the correctness of 256 the model based on the first sample subset is evaluated by the 257 number of inliers. These two steps are iteratively repeated until 258 the model has the highest level of consistency (that is, the highest 259 number of inliers). 260

For a RANSAC algorithm, there are three main parameters: 261 the sample number of the subset, the inlier judgment threshold 262 and the maximum iteration [15]. The sample number of the sub-263 set depends on the minimum number of data elements required 264 for model estimation. The inlier judgment threshold is generally 265 set according to the desired confidence level. RANSAC is a 266 nondeterministic algorithm in the sense that it produces a reason-267 able result only with a certain probability, with this probability 268 increasing as more iterations are allowed. However, iterating 269 through all subsets is too time-consuming for a large sample, 270

so it is necessary to set an iteration threshold to improve the 271 algorithm efficiency. 272

B. Subset Selection in TC-GNSS/INS Solution 273

The number of subset samples is the minimum number of 274 data elements required for model estimation, and it refers to 275 the minimum number of satellites for GNSS positioning in 276 tightly coupled GNSS RTK/INS integration. In the conventional 277 GNSS positioning solution, it is generally believed that at least 4 satellites are required to estimate three-dimensional position 279 and the receiver clock error. 280

Compared with the conventional GNSS solution, tightly 281 coupled integration increases the INS assistance; therefore, 4 282 satellites are not necessary. We have previously analyzed the 283 auxiliary effect of different numbers of satellites on the tightly 284 coupled integration, and it will not be repeated in this article. Our 285 preliminary work based on multiple field tests results show that 286 2 satellites with good geometric distributions can improve the 287 integrated navigation accuracy. Therefore, the number of subset 288 samples is 2 satellites in this article. This is also the advantage 289 of the proposed method over the conventional GNSS solution. 290

C. Inlier Judgment in TC-GNSS/INS Solution

The inlier judgment is based on whether the observed GNSS 292 range information is consistent with the model formed by the 293 current subset. Here, the integrated navigation results, which are 294 obtained from the tightly coupled integration solution assisted 295 by the 2 satellites in the subset, can be used to perform inverse 296 computation of the range observation. The derived range and 297 the real observed range outside the subset are used to construct 298 the range residual that is the basis of the inlier judgment. The 299 following analysis will illustrate the calculation process of the 300 range residual and its standard deviation with the carrier phase 301 observation as an example. 302

The between-receiver single-difference carrier phase observation $\tilde{\varphi}_{br}^{s}$ is given in (4); here, it is rewritten as 304

$$\tilde{\varphi}_{br}^{s} = \frac{1}{\lambda}\rho_{br}^{s} + \frac{1}{\lambda}T_{bias}^{sys} + \Delta N \tag{8}$$

291

The derived single-difference carrier phase $\hat{\varphi}^s_{br}$ can be expressed by 305

$$\hat{\varphi}_{br}^{s} = \frac{1}{\lambda}\hat{\rho}_{br}^{s} + \frac{1}{\lambda}\hat{T}_{bias}^{sys} + \Delta\hat{N}$$
(9)

where $\hat{\rho}_{br}^s$ and \hat{T}_{bias}^{sys} can be obtained from the model parameters. 307 However, $\Delta \hat{N}$ is unknown, because the estimated results based 308 on the subset only include the single-difference ambiguity of 309 the selected 2 satellites, and the ambiguity of the remaining 310 satellites outside the subset is presently unknown. Therefore, it 311 is necessary to eliminate the single-difference ambiguity. 312

In general, the ambiguity remains the same for two adjacent 313 epochs, so it can be removed using the between-epoch difference 314 to yield the following expression. 315

$$\nabla\Delta\tilde{\varphi}^{s} = \tilde{\varphi}^{s}_{br}\left(t_{2}\right) - \tilde{\varphi}^{s}_{br}\left(t_{1}\right) \tag{10}$$

where $\nabla \Delta \tilde{\varphi}^s$ is the double-difference range observation, and the double difference is a between-epoch single difference of a between-receiver single difference; t_1 and t_2 are two adjacent epochs.

According to (9), the derived double-difference carrier phase $abla \Delta \hat{\varphi}^s$ can be expressed by

$$\nabla\Delta\hat{\varphi}^{s} = \frac{1}{\lambda} \left(\hat{\rho}_{br}^{s}(t_{2}) + \hat{T}_{bias}^{sys}(t_{2}) \right) - \frac{1}{\lambda} \left(\hat{\rho}_{br}^{s}(t_{1}) + \hat{T}_{bias}^{sys}(t_{1}) \right)$$
(11)

Combining (10) and (11) yields the double-difference carrier phase residual as

$$\delta\varphi^s = \nabla\Delta\hat{\varphi}^s - \nabla\Delta\hat{\varphi}^s \tag{12}$$

The double-difference carrier phase residual is the basic parameter used for the inlier judgment, and the corresponding variance σ^2 (σ is the corresponding standard deviation) can be written as,

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 \tag{13}$$

where σ_1^2 represents the variance of the derived doubledifference carrier phase $\nabla\Delta\hat{\varphi}^s$ and σ_2^2 represents the variance of the observed double-difference carrier phase $\nabla\Delta\hat{\varphi}^s$.

According to (11), the variance of the derived doubledifference carrier phase can be expressed by

$$\sigma_1^2 = \frac{1}{\lambda^2} \left(\sigma_{\rho 2}^2 + \sigma_{t2}^2 + \sigma_{\rho 1}^2 + \sigma_{t1}^2 \right)$$
(14)

where $\sigma_{\rho 1}^2$ and $\sigma_{\rho 2}^2$ are the variances of the range derived from the INS at the adjacent two epochs; σ_{t1}^2 and σ_{t2}^2 are the variances of the single-difference clock errors at the adjacent two epochs, respectively, and they can be obtained from the state variance matrix of the Kalman filter.

Through linearization expansion and spatial transformation, and ignoring the small effect of the covariance $D_{r\phi}$ between position and attitude, the variance σ_{ρ}^2 of the range derived from the INS can be written as

$$\sigma_{\rho}^{2} = H_{r}C_{n}^{e}\left(D_{r,IMU}^{n} + H_{\phi}D_{\phi}H_{\phi}^{\mathrm{T}}\right)C_{n}^{e}{}^{\mathrm{T}}H_{r}^{T} \qquad (15)$$

where H_r is the linearized matrix of the single-difference range; C_n^e is the direction cosine matrix from the navigation frame to the Earth frame; H_{ϕ} is the designed attitude matrix; $D_{r,IMU}^n$ is the variance matrix of the INS position error; and D_{ϕ} is the variance matrix of the INS attitude error.

According to (4) and (10), σ_2^2 can be calculated by the following equation.

$$\sigma_2^2 = n_{r2} + n_{b2} + n_{r1} + n_{b1} \tag{16}$$

where n_{r1} and n_{r2} are the variances of the measurement noise of the rover station at the two adjacent epochs; n_{b1} and n_{b2} are the variances of the measurement noise of the base station at the two adjacent epochs.

Theoretically, the constructed double-difference carrier phase residual is normally distributed, so the criterion of the inlier and the outlier can be written as,

$$\begin{cases} \delta \varphi^s < T1, \quad s \in inlier\\ \delta \varphi^s \ge T1, \quad s \in outlier \end{cases}$$
(17)

where the threshold value (denoted by T1 in Section III), which depends on the level of the required confidence, can be set as a multiple of the standard deviation of the residual. For example, the confidence level is 99.73% when the threshold is set to 3σ . 359

D. Subset Iteration in TC-GNSS/INS Solution 360

Different from the computer vision, the maximum number 361 of iterations need not be limited when the RANSAC algorithm 362 is applied to fault detection and exclusion of tightly coupled 363 integration. This is because the number of visible satellites is 364 limited, resulting in a small subset size with only 2 satellites. 365 However, subset construction requires consideration of the geo-366 metric distribution of these 2 satellites to ensure the accuracy of 367 the tightly coupled integration. From experience, it is better that 368 the azimuth difference between the 2 selected satellites generally 369 ranges from 60° to 120°. 370

On the basis of the conventional RANSAC algorithm as 371 described in Section III, the proposed algorithm adds the global 372 proportion statistics of faults to ensure detection reliability. 373 The global proportion statistics of faults consist of two steps: 374 recording the number of satellites classified as faults during 375 subset iteration and calculating the percentage of faults for each 376 satellite. Note that when satellites are included in the subset, 377 they are not classified as faults, and there are differences in the 378 number of satellites involved in constructing subsets. To exclude 379 the influence of these factors, the global proportion statistics of 380 the faults can be expressed as 381

$$R_a = \frac{OC}{SN - ISN} \tag{18}$$

where R_a represents the ratio of faults; OC is the number of 382 satellites classified as faults; SN is the total number of subsets 383 with 2 satellites in the current epoch; and ISN is the number of 384 detected satellites involved in the subset. 385

$$\begin{cases} R_a < T3, & s \in fault - free \\ R_a \ge T3, & s \in fault \end{cases}$$
(19)

In Section IV-B, we will delve into the impact of threshold 391 T3 value on fault detection. It's important to note that unlike 392 T1, which can be stochastically related to the probability of 393 false positives, T3 is determined by balancing the recall and 394 precision of fault detection. 395

E. Algorithm Framework of RANSAC-Based FDE in TC 396 GNSS/INS Solution 397

Fig. 2 shows the flow of the RANSAC-based fault detection398and exclusion of the tightly coupled integration. Block ① on399the left shows the operations performed for each subset. First,400tightly coupled integration based on the 2 selected satellites is401conducted to obtain the integrated solution. Second, the double-402difference residual and the corresponding standard deviation of403



Fig. 2. Flow chart of RANSAC-based fault detection and exclusion of tightly coupled GNSS/INS integration.

satellites outside the subset are calculated. Finally, the residual of each satellite is compared with the preset threshold T1, and the number of satellites classified as faults is recorded. Here, threshold T1, which is not constant, is related to the standard deviation of the double-difference residual.

The algorithm flow continues to the proposed global fault 409 proportion statistics procedure shown in block (2) on the right 410 when all subsets have been iterated and processed. The ratio 411 412 of satellites classified as faults is calculated and compared with preset threshold T3 for reliable fault detection and exclusion. 413 In general, the smaller the threshold is, the easier it is to detect 414 satellite observation faults, but the possibility of false positives is 415 also higher. Conversely, the larger the threshold is, the more dif-416 ficult it is to detect faults, but the possibility of false positives is 417 also lower. Hence, a reasonable threshold is a key parameter 418 to ensure the effectiveness of fault detection. The effect of the 419 threshold on the detection performance will be analyzed in the 420 following section. 421

422 IV. EFFECT ANALYSIS OF PARAMETERS ON RANSAC-BASED 423 FAULT DETECTION PERFORMANCE

In this section, we predominantly examine the influence of two parameters on the effectiveness of the proposed RANSACbased fault detection algorithm: the threshold T3 and the quantity of faulty satellites. Our goal is to provide a quantitative analysis of the algorithm's performance. To achieve this, we

TABLE I Optimized Specifications of ICM20602

Sensor	Parameters	ICM20602
Gyro	In-run bias instability (°/hr) White noise (°/ \sqrt{hr})	50 0.24
Accel.	In-run bias instability (ug) White noise (m/s/ √ hr)	250 0.24

introduced artificial cycle slips (an example of step errors) with 429 varying magnitudes into the raw carrier phase observations 430 collected from a vehicle-mounted rover receiver in an open-sky 431 environment. The magnitudes are in order as follows: 0.5 cycles, 432 1.0 cycle, 2.0 cycles and 3.0 cycles, denoted as 0.5c, 1c, 2c, and 433 3c, respectively. The number of visible satellites was limited to 434 12. A low-end MEMS grade GNSS/INS system with ICM20602 435 from TDK InvenSense was used for processing and analysis. 436 Table I lists the optimized specifications of the MEMS IMU. 437

A. Performance Evaluation Metrics

For the statistical classification problem, a confusion matrix is a specific table layout that allows visualization of the performance of an algorithm [27]. For binary classification, the scheme of the confusion matrix is shown in Table II. Each row of the matrix represents the instances in an actual class, while each column represents the instances in a predicted class. The

	TABLE II
CONFUSI	ON MATRIX FOR BINARY CLASSIFICATION

Total		Predicted class		
		Positive	Negative	
Actual class	Positive	True Positive (TP)	False Negative (FN)	
	Negative	False Positive (FP)	True Negative (TN)	

confusing matrix can make it easy to see whether the system isconfusing two classes.

447 The confusion matrix for binary classification shown in Table II presents four classification results: "TP" is the true 448 positive value, which is the number of positive observations 449 classified correctly; "TN" is the true negative value, which is 450 the number of negative observations classified correctly; "FP" 451 452 is the false positive value, which is the number of actual negative observations classified as positive; and "FN" is the false nega-453 tive value, which is the number of actual positive observations 454 classified as negative. 455

In essence, fault detection is a binary classification problem,
so the performance evaluation metrics of the fault detection
algorithm were borrowed from the terminology and derivations
of a confusion matrix [27]. The calculations of the performance
metrics, including the accuracy (ACC), precision (PRE), recall
(REC) and F-score (Fs) values, are made according to (20)–(23).

$$ACC = \frac{TP + TN}{TP + TN + FP + FN}$$
(20)

$$PRE = \frac{TP}{FP + TP} \tag{21}$$

$$REC = \frac{TP}{FN + TP}$$
(22)

$$Fs = 2 \times \frac{PRE \times REC}{PRE + REC}$$
(23)

It should be noted that the ACC reflects the probability of 462 observations classified correctly, but it can be misleading if 463 used with imbalanced datasets. The PRE represents the ratio 464 of the detected actual negative observations relative to those 465 classified as negative observations, and the lower the PRE is, 466 the higher the false detection rate. The REC represents the ratio 467 of the detected actual negative observations relative to all actual 468 negative observations, and the lower the REC is, the higher the 469 missed detection rate. The Fs value is the harmonic mean of the 470 PRE and REC. 471

In fault detection, missed detection can lead to faults being 472 included in the integrated navigation solution and producing 473 incorrect results, while false detection can result in accurate 474 observations not being used to reduce the integrated navigation 475 accuracy. Therefore, while guaranteeing a certain REC level, the 476 PRE magnitude should be considered. In the following, we will 477 utilize these two metrics to analyze the effect of parameters on 478 detection performance, and the analysis results are displayed in 479 480 the form of a percentage of performance metrics.



Fig. 3. Performance metric curves representing the effect of threshold T3 on RANSAC-based fault detection with different numbers of artificial cycle slips.

B. Effect of Thresholds on Detection Performance

There are two thresholds, T1 and T3, that need to be set in the proposed fault detection algorithm. The setting of threshold T1 will not be discussed in detail, and the judgment is mainly based on the residual sequence of the double-difference carrier phase. Here, the threshold T1 is set to 1σ in order to detect small cycle slips (e.g., 0.5-cycle) and effectively capture larger cycle slips (e.g., >1-cycle). 488

Fig. 3 shows the REC and PRE representing the effect of 489 threshold T3 on RANSAC-based fault detection with different 490 artificial cycle slips. There are 6 satellites with faults, and 491 threshold T3 varies from 0.2 to 0.8. Considering the REC, the 492 value with 0.5 cycle slips is the lowest under the same threshold 493 T3, which indicates that the detection of 0.5 cycle slips is the 494 most difficult. The REC values of all cycle slips decrease as 495 threshold T3 increases, which indicates that the missed detection 496 rate increases as threshold T3 increases. 497

Considering the PRE, there is less variation in the value with 498 0.5 cycle slips when threshold T3 is changed, and the PRE value 499 can be basically controlled above 80%. The PRE value of $1 \sim 3$ 500 cycle slips is less than 60% when threshold T3 is less than 0.5 to 501 increase the false detection rate. If a small cycle slip (e.g., less 502 than (0.5c) is the main error, the threshold T3 can be set to (0.4). 503 If the large cycle slip (e.g., larger than 1.0c) is the main error, 504 the threshold T3 should be set to 0.7. The threshold T3 can be 505 set to 0.6 when taking into account cycle slips of $0.5c \sim 3.0c$. 506

C. Effect of Fault Number on Detection Performance

Fig. 4 shows the performance metric curves representing the 508 effect of the number of faulty satellites on the RANSAC-based fault detection with different artificial cycle slips. The total number of visible satellites is 12, and the number of satellites 511 with artificial faults is $1 \sim 8$. The REC value of the detection 512

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Fig. 4. Performance metrics curves representing the effect of the number of faulty satellites on the RANSAC-based fault detection with different numbers of artificial cycle slips.

algorithm decreases as the number of faulty satellites increases,
especially for 0.5 cycle slips. The REC value with 0.5 cycle
slips basically remains above 90% when the number of faulty
satellites is less than 4.

Different from the REC value, the PRE value does not al-517 ways decrease as the number of faulty satellites increases. The 518 fluctuation of the PRE curve with 0.5 cycle slips is small, and 519 the overall performance decreases with an increasing number 520 of faulty satellites, while the PRE curves with $1 \sim 3$ cycle slips 521 show a trend of first decreasing and then increasing. The larger 522 the threshold T3 is, the greater the number of faulty satellites at 523 the minimum value of the curve. The number of faulty satellites 524 corresponding to the minimum value of the curve is 4 and 6 525 when the threshold T3 is 0.4 and 0.6, respectively; the number 526 of faulty satellites is 8 when T3 is 0.7, which makes the curve 527 show a monotonically decreasing trend. 528

For the special trends in the PRE curve, since the total number of satellites is fixed, an increase in the number of faulty satellites results in a decrease in the number of normal satellites. At this time, the detection algorithm has the possibility of false detection, but the number of satellites that can be classified decreases, so the PRE value increases instead.

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V. TESTS AND RESULTS

This section presents an analysis of positioning performance in typical urban scenarios and provides statistics from multiple tests conducted in urban environments. Section A focuses on navigation performance in various scenarios, while Section B



Fig. 5. Land vehicle test trajectory segmented with letters (To the left is north, generated by google earth).

TABLE III Scenario Descriptions of Different Road Segments

Road segment	Scenario description	Time Proportion
AB	Crossing an urban canyon, GNSS signals were blocked by buildings, and the number of visible satellites was approximately 6.	29.7%
BC	Under a viaduct, the number of visible satellites is less than 6.	15.8%
CD	On a viaduct, and GNSS signals were blocked for 49 seconds at the end of the viaduct due to the noise barrier.	18.2%
DE	Tunnel, there is no GNSS signal.	24.2%
EF	Boulevard, the number of visible satellites is more than 6.	12.1%

discusses the effectiveness and availability of the proposed 540 RANSAC-based fault detection and exclusion method. 541

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A. Performance Analysis of Typical Urban Scenarios

To further explore the comprehensive performance of the 543 RANSAC-based fault detection of tightly coupled integration in 544 typical urban scenarios, a land vehicle test covering buildings, 545 tunnels, and viaducts was conducted in Wuhan city. Fig. 5 shows 546 the test trajectory, and the trajectory distance is approximately 547 4.5 km. The detailed scenario descriptions of different road 548 segments marked with letters are listed in Table III, and the 549 vehicle speed is low in the downtown such as segment AB and 550 BC. 551

Fig. 6 shows the installation of the equipment used for the field 552 land vehicle test. The INSProbe is a MEMS grade GNSS/INS 553 integrated system with ICM20602 from TDK InvenSense, and 554 a NovAtel OEM718D card is used for GNSS data acquisition. 555 The POS620 is a navigation grade GNSS/INS integrated system 556 with a high-grade fiber optic gyro (FOG), and its postprocessing 557 smoothed results serve as the reference truth for data analysis. 558 The specifications of these two IMUs are listed in Table IV. 559

Various data processing modes are employed to evaluate the viability of the proposed fault detection method in urban areas. For a detailed description of the data processing mode, see Table V, which outlines the implementation of an innovationbased fault detection method utilizing the tightly coupled GNSS/INS integration. 560 561 562 563 564 565



Fig. 6. Installation of the equipment used for the field land vehicle test.

TABLE IV SPECIFICATIONS OF IMUS

Sensor	Parameters	ICM20602	POS620 (reference)
Gyro	In-run bias instability (°/hr)	50	0.03
	White noise (°/ \sqrt{hr})	0.24	0.003
Accel.	In-run bias instability (ug)	250	15
	White noise (m/s/ \sqrt{hr})	0.24	0.03

TABLE V DATA PROCESSING MODE DESCRIPTIONS

F	Process Mode	Mode Abbreviation	Description
	GNSS RTK	RTK	Forward processing of single frequency GPS/BDS data
GNSS/INS integration	Loosely coupled	LC	Forward filter of RTK/INS loosely coupled integration with robust estimation based on innovation
	Tightly coupled	TC1	Forward filter of RTK/INS tightly coupled integration with robust estimation based on innovation
	Tightly coupled with RANSAC	TC2	Forward filter of RTK/INS tightly coupled integration with RANSAC-based fault detection

Fig. 7 shows the position error of the different processing 566 modes, and the number of satellites, including visible satellites, 567 satellites with cycle slip, and satellites rejected. The GNSS 568 interruption interval is marked on the horizontal axis with a 569 yellow block. Overall, the TC2 mode boasts good position 570 accuracy, particularly in challenging situations, and is supported 571 by the proposed RANSAC-based method for fault detection. 572 The positioning performance is analyzed segment by segment 573 to show the characteristics of different processing modes in 574 different scenarios. 575

Before segment AB, the RTK mode can maintain a fixed solution. During segment AB, the position accuracy and continuity
of the RTK mode are significantly reduced as the number of
satellites gradually decreases, and the position accuracy of the
LC mode is affected by the GNSS positioning performance.

TABLE VI Performance Evaluation Metrics Descriptions

Metric	Metric Descriptions		
Max	Maximum of the absolute value of the navigation error.		
RMS	Root mean square of the navigation error.		
CDF95	Error value corresponding to the cumulative distribution function with 95%.		
Fixed rate	Proportion of epochs with ambiguity correctly fixed.		
Valid rate	Proportion of epochs with position error is less than 5.0 m.		
Success rate	Proportion of epochs with correct positioning.		

For the TC1 and TC2 modes, there is no obvious difference in 581 cycle slip detection and satellite rejection, and the corresponding 582 position accuracy can be controlled within 2.0 m even when 583 there are fewer than 4 satellites. During segment BC, the number 584 of visible satellites is approximately $3 \sim 4$, which is caused by 585 severe GNSS signal occlusion caused by the viaduct. Although 586 the position accuracy of all modes is poor, that of the TC1 and 587 TC2 modes can be controlled within 5.0 m and has a relatively 588 good position accuracy compared with the RTK and LC modes. 589

During segment CD, there is a difference in cycle slip detec-590 tion and satellite rejection for the TC1 and TC2 modes, and the 591 RANSAC-based fault detection method guarantees the tightly 592 coupled integrated position accuracy of the TC2 mode in the 593 challenging scenario. The correct fault detection of the TC2 594 mode before entering the tunnel reduces the position error diver-595 gence level compared with the TC1 mode. The GNSS signals 596 of segment DE are interrupted for approximately 3 minutes, 597 and the horizontal position error of the TC2 mode diverges to 598 approximately 10 m, while the horizontal position error of the 599 TC1 mode reaches 30 m. 600

During segment EF, a large number of fault-free satellites 601 were mistakenly eliminated in the TC1 mode, and a long time 602 was required to achieve the convergence of position error. Conversely, the TC2 mode completed the rapid convergence of 604 position error because of the RANSAC-based fault detection 605 method, which effectively controlled the false detection rate and 606 the missed detection rate. 607

In a typical environment, the RTK and LC modes can experi-608 ence significant disruption to their positioning performance from 609 external environmental disturbances. However, the TC mode 610 has the capability to leverage the raw GNSS observations to 611 achieve a reliable GNSS/INS integration solution even when the 612 number of satellites is less than four. Notably, the TC2 mode has 613 implemented a RANSAC-based fault detection mechanism to 614 further enhance positioning accuracy in challenging scenarios. 615

In addition, we also used statistical results for performance 616 evaluation, and the performance evaluation metrics are defined 617 as shown in Table VI. 618

Fig. 8 shows the performance evaluation metrics of the dif-619 ferent processing modes. The position accuracy represented by 620 the Max, RMS and CDF95 of the TC2 mode is significantly 621 better than that of the TC1 and LC modes. Since faults are not 622 correctly detected and eliminated before and after the tunnel, 623 the north position error of the TC1 mode is larger than that 624 of the LC mode. The success rate of the RTK mode is less 625 than 50% because there is frequent GNSS signal interruption 626



Fig. 7. Position error of the different processing mode and the number of satellites.



Fig. 8. Performance evaluation metrics of the different processing modes.

caused by the external environment. Although the LC mode can
maintain continuous positioning, the corresponding valid rate is
only 33%. The valid rate of the two tightly coupled modes is
more than 60%, and compared with the TC1 mode, the valid
rate and fixed rate of the TC2 mode are increased by 29% and
19%, respectively.



Fig. 9. Performance statistics of multiple tests in urban environment.

633 B. Performance Statistics of Multiple Urban Environments

Multiple land vehicle tests were conducted in a complex urban environment to evaluate the feasibility of the RANSAC-based fault detection in tightly coupled integration. Here, the total time length of field test is approximately 7 hours and the environmental conditions include the downtown, campus, city
tunnel and viaduct etc. Fig. 9 presents the statistics obtained
from these tests. Overall, the maximum position errors and the
CDF95 values of the TC2 mode are smaller than those of the
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TC1 mode, and the fixed rate and valid rate are significantly 642 higher than those of the TC1 mode. The proposed RANSAC-643 based fault detection algorithm significantly improved the north 644 645 and east position accuracy (in terms of CDF95) of the tightly coupled mode in the comprehensive scenario, with an average 646 increase of 45% and 42% respectively. This indicates that the 647 positioning performance of the TC2 mode has been enhanced by 648 the RANSAC-based fault detection algorithm in complex urban 649 environments. 650

651 However, in relation to data 2, the TC2 mode displays smaller maximum position errors and CDF95 values compared to the 652 TC1 mode, yet its fixed rate remains lower. This discrepancy 653 654 suggests that the proposed fault detection algorithm has yielded a high false positive rate, incorrectly classifying normal GNSS 655 observations as faults. The reason behind the unsatisfactory PRE 656 value can be attributed to the greater emphasis given to the 657 REC value for ensuring position error level. This also highlights 658 the flaws in the threshold setting approach of the proposed 659 algorithm. Fixed thresholds may not be suitable for all scenarios, 660 thereby rendering the algorithm inaccurate. 661

Based on the above analysis of land vehicle tests, it can be seen the TC2 mode can provide navigation information with high performance due to RANSAC-based fault detection and exclusion, and it is better that the thresholds should be adaptively adjusted to ensure the applicability of the proposed algorithm.

VI. CONCLUSION

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This work draws on the application of the RANSAC algorithm 669 for GNSS fault detection, and proposes a RANSAC-based fault 670 detection and exclusion of a tightly coupled GNSS RTK/INS 671 integration for a high-accuracy positioning solution in urban 672 environments. The between-receiver single-difference tightly 673 coupled mode was applied to fully utilize valid GNSS obser-674 vations. The characteristics of RANSAC-based algorithm for 675 tightly coupled integration were analyzed from the aspects of 676 subset selection, inlier judgment, subset iteration and so on. 677 A fault global proportion statistics was extended to the typical 678 RANSAC algorithm to enhance the detection reliability. 679

Simulation tests, where artificial cycle slips of different mag-680 nitudes were inserted into raw GNSS observations in an open-681 sky environment, were conducted to analyze the performance 682 of the proposed RANSAC-based fault detection algorithm. The 683 test results show that the proposed algorithm can effectively 684 detect small faults and multiple faults, and the detection rates 685 for 0.5c and $1c \sim 3c$ slips were approximately 70% and 90%, 686 respectively. Furthermore, land vehicle tests that included typi-687 cal scenarios in complex urban environments were conducted 688 to further investigate the comprehensive performance of the 689 proposed algorithm. The results indicate that the tightly coupled 690 mode was more suitable for changeable GNSS environments 691 compared to the loosely coupled mode; and with the help of the 692 proposed RANSAC-based fault detection algorithm, the north 693 and east position accuracy (in terms of CDF95) of the tightly 694 coupled mode in the comprehensive scenario was improved by 695 696 an average of 45% and 42%.

The proposed RANSAC-based fault detection algorithm can be further applied to multi-sensor information fusion, and guarantee a high level of accuracy and reliability in the positioning solution in harsh urban environments. Our subsequent work will thoroughly compare with the existing methods and optimize the threshold setting scheme to ensure the superiority and universality of the proposed algorithm. 703

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